OPTIC EMBEDS INTO THE INT CONSTRUCTION

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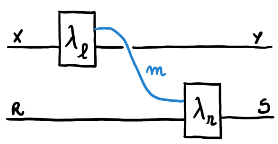
On this note we use Riley's definition of **Optic** [Ril18, definition 2.0.1]. Let $\mathcal{C}$ be a symmetric monoidal category, an optic $\binom{X}{S} \to \binom{Y}{R}$ is given by an element of the set

$$\int^{M\in\mathcal{C}} \mathcal{C}(X, M \otimes Y) \times \mathcal{C}(M \otimes R, S)$$

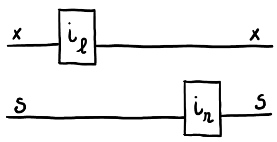
whose elements are pairs of morphisms $\langle l \mid r\rangle$ for $l\colon X \to M \otimes Y$ and $r\colon M \otimes R \to S$ quotiented by the equivalence relation generated by $\langle (f \otimes Y) \circ l \mid r\rangle \sim \langle l \mid r \circ (f \otimes R)\rangle$ for any $f\colon M \to N$.

Proposition 1. *Let $\mathcal{C}$ be a symmetric traced category* [Sel10, §5.7]*. There is a strict monoidal functor $-^*\colon$ **Optic**$(\mathcal{C}) \to$ **Int**$(\mathcal{C})$, which is the identity on objects and takes an optic $\lambda\colon \binom{X}{S} \to \binom{Y}{R}$ given by $\langle \lambda_l \mid \lambda_r\rangle$ to*



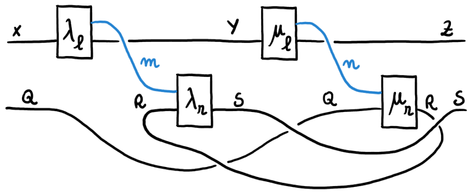
where this morphism is well-defined because it is dinatural on the residual *of the optic.*

Proof. The identity morphism $\binom{X}{S} \to \binom{X}{S}$ of **Optic**$(\mathcal{C})$ is sent to



which is equal to the identity on $X \times S$. This is the identity morphism $\binom{X}{S} \to \binom{X}{S}$ of **Int**$(\mathcal{C})$.

Next, consider morphisms $\lambda\colon \binom{X}{S} \to \binom{Y}{R}$ and $\mu\colon \binom{Y}{R} \to \binom{Z}{Q}$ in **Optic**$(\mathcal{C})$. If we compose them in **Int**$(\mathcal{C})$ we obtain the morphism $\mu^* \circ \lambda^*$ with string diagram



Using the symmetric traced structure of $\mathcal{C}$, we can transform this into the morphism $(\mu\circ\lambda)^*$ with the following manipulations of the string diagram, which yield the equality of

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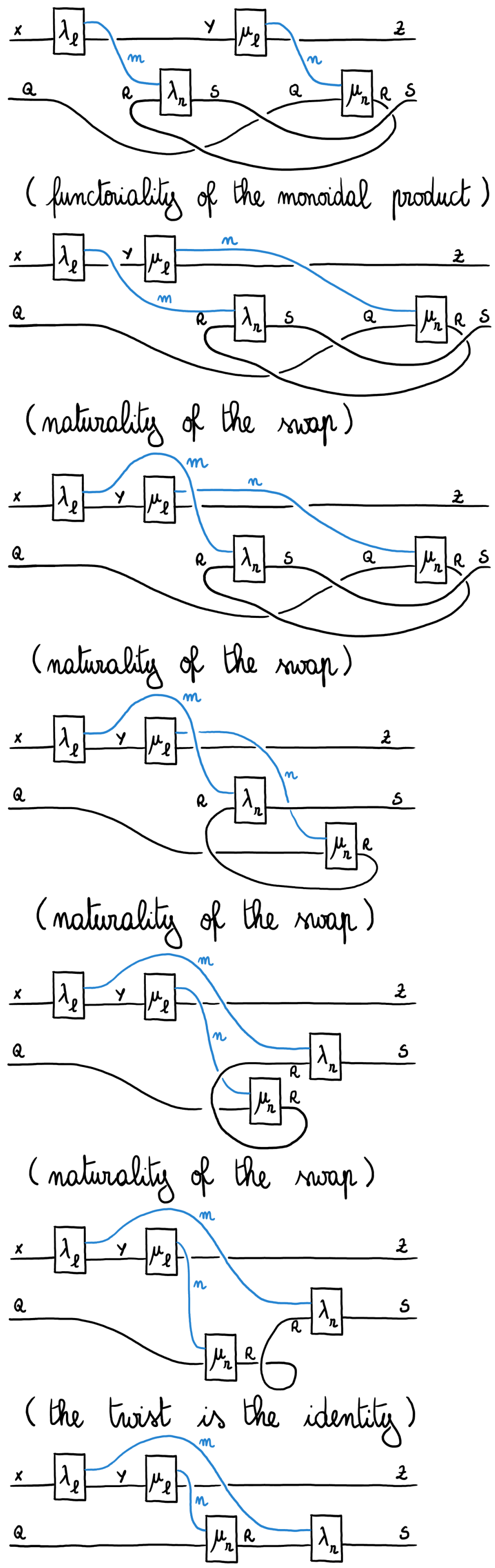
the two morphisms via the coherence theorem for traced symmetric monoidal categories [Sel10, theorem 5.22].



Finally, it can be seen that the functor is strict monoidal. □

The embedding in Proposition 1 particularizes to the embedding by Hedges [Hed19] for the case of traced cartesian categories. In that case, the first component of the optic can always be written as the comultiplication of the cartesian structure.

References

[Hed19] Jules Hedges. The game semantics of game theory. *CoRR*, abs/1904.11287, 2019.

[Ril18] Mitchell Riley. Categories of optics. *arXiv preprint arXiv:1809.00738*, 2018.

[Sel10] Peter Selinger. A survey of graphical languages for monoidal categories. In *New structures for physics*, pages 289–355. Springer, 2010.