Universal Properties of Asynchronous Message Passing

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Abstract.

Message passing (specifically, *multi-party* and *asynchronous* message passing) is the most common technique in computer science that allows different systems to interact and coordinate. There is a vast literature about different message passing models (actors, π-calculus, event structures, or game semantics); but, arguably, we lack a universal characterization of the fundamental combinatorial structures underlying message passing. What universal properties characterize message passing?

In this talk, I shall introduce a minimalistic logic for message-passing protocols informed by category theory: its derivations form the free polarized normal monoidal symmetric multicategory over a set of objects. Pleasantly, arguing that message-passing can be universally implemented over any process can be now formalized as an adjunction between symmetric monoidal categories and algebras for this universally characterized message-passing multicategory.

We will investigate how category theory helps us justify this choice and characterize the two essential aspects of message passing: the interleaving of events (via *normal duoidal categories*), and the duality between sending and receiving (via *polarization*, the left adjoint to picking adjunctions).

Normal duoidal categories, pseudomonoids in the 2-category of monoidal categories and lax monoidal functors whose two units coincide that were introduced for the study of Hopf algebras, have been repeatedly linked to notions of causal dependence and admissible commutativity (esp. in the work of Garner and López Franco). Already back in 1981, Grabowski characterized some expressible posets arising from duoidal expressions; Gischer, in 1998, recognized the lax interchange of normal duoidal categories as a sumsumption of posets. In this talk, we shall develop a non-representable counterpart: normal monoidal symmetric multicategories, which follow the monoidal multicategories of López Franco and Vasilakopoulou. We will recall how *shufflings*, permutations of the joint elements of multiple lists that leave invariant the internal ordering of each list, form the free normal monoidal symmetric multicategory.

Polarization is left adjoint to picking dualities: in particular, the dualities of a normal monoidal symmetric multicategory have a left adjoint creating *freely polarized* normal monoidal symmetric multicategories. Polar shufflings, our main structure, will arise as the free normal monoidal symmetric multicategory over a set of objects. Polar shufflings are equipped with a natural presheaf represented by the empty list: this means we can apply Shulman's polycategorical Chu-Dialectica construction to recover a star-polycategory of polar shufflings.

We will use elementary category theory to justify our constructions: no knowledge of message passing systems will be assumed nor necessary. We will briefly introduce the basic theory of normal duoidal categories, normal monoidal multicategories, and the polycategorical Chu construction.

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